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Theoretical and Simulated Evaluation of 16-DAPSK in Mobile Fading Channels

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Abstract: *Various 16-level modulation schemes have been investigated with particular emphasis being applied to differential 16-APSK. The performance of this technique has been optimised in Rayleigh fading taking into account both AWGN and Random FM noise. The impact of diversity has also been investigated and results compared with those obtained through extensive computer simulation.*

1 Introduction

There is a growing demand for future mobile systems to provide not only speech but also integrated data communications. As the complexity of these systems increase, the desire for higher data rates will produce an obvious need for spectrally efficient digital transmission.

Fig.1 shows the constellation diagrams for three 16-level modulation schemes. Whilst square 16-QAM is clearly optimal in white Gaussian noise, in a mobile channel problems can arise due to carrier *false locking*. To illustrate this problem Fig.2 shows the average error in a carrier recovery loop as the received constellation is rotated relative to the receiver's reference. As would be expected, at multiples of 90° the loop error falls to zero due to the symmetry of the square constellation. These ambiguous lock positions can be overcome through the use of differential quadrant encoding. However, for rotations of 26.6° , 36.9° , 53.1° and 63.4° distinct local minima may occur and these can result in false locking. Since the received and reference constellations are not aligned during these false lock positions *hard* differential detection can no longer be applied.

The above problem may be overcome by either masking symbols on the middle ring [1] or using *soft* differential detection. The first approach removes the phase non-uniformity associated with the square 16-QAM constellation whilst the second technique removes any de-

pendency on the absolute value of the phase. Unfortunately, in a mobile fading channel the required symbol masking is difficult to reliably achieve due to the rapid variations in signal envelope. Alternatively, using soft differential detection produces a large number of amplitude and phase transients and this results in a system that performs poorly in noise [2].

Rather than using square 16-QAM, various alternative constellations have been proposed over the years. Circular 16-QAM represents one of the more promising configurations and is shown in Fig.1(ii) [3]. The use of just 8 uniformly spaced phases removes any possibility of false locking and also allows the use of coherent reception. However, for fully differential detection this constellation also suffers since it generates a large number of amplitude and phase transients. To achieve efficient differential detection, 16-APSK may be used [2] [4] [5] - see Fig.1(iii). This approach allows the amplitude and phase bits to be received separately using independent differential receivers. This property allows the 16-APSK constellation to be efficiently received using differential detection.

This paper analyses the performance of fully differential 16-APSK (16-DAPSK) and also shows that its performance may be improved through the addition of diversity. Since the amplitude and phase encoded bits are detected independently it follows that separate diversity techniques must be applied. In this analysis square-law and maximum-ratio combining have been proposed for the amplitude and phase encoded bits respectively.

2 System Modelling

The mathematical model of the system assumes that there are L diversity branches, each carrying identical information. The fading on each branch is assumed to be frequency flat with envelope statistics that follow

a Rayleigh distribution. For each of the branches the fading process is also assumed to be mutually statistically independent.

The equivalent lowpass transmitted signal $s(t)$ can be written as below:

$$s(t) = A(t) \exp[j\theta(t)] \quad 0 \leq t \leq T_s \quad (1)$$

where $A(t)$ represents the amplitude of $s(t)$, which takes one of two values, A or βA , depending on the particular data symbol being sent (β represents the ring ratio for the constellation). The information phase $\theta(t)$ takes values in the set $2\pi(i-1)/8$, $i = 1, 2, \dots, 8$. A rectangular pulse shape with duration T_s is assumed, where T_s represents the symbol period.

For a frequency flat fading channel, the equivalent low-pass received signal for any diversity branch can be written in the following form:

$$r_k(t) = g_k(t)s(t) + z_k(t) \quad \begin{matrix} 0 \leq t \leq T_s \\ k = 1, 2, \dots, L \end{matrix} \quad (2)$$

where, for the k^{th} diversity branch, $g_k(t)$ represents the zero mean complex Gaussian fading process and $z_k(t)$ the complex additive white Gaussian noise. The auto-correlation function for $g_k(t)$ and $z_k(t)$ are stated below:

$$R_g(\tau) = E[g(t)g^*(t-\tau)] = \mu_g(\tau)R_g(0) \quad (3)$$

$$R_z(\tau) = E[z(t)z^*(t-\tau)] = 2N_o\delta(\tau) \quad (4)$$

where $R_g(0)$ is the power of the fading process, $\mu_g(\tau)$ is the auto-correlation coefficient of the fading process and N_o is the normalised power spectral density. For mobile applications $\mu_g(\tau)$ may be defined as $J_0(2\pi f_d\tau)$ where $J_0(\cdot)$ represents the Bessel function of order zero and f_d the maximum Doppler frequency. The received symbol energy is $A^2(t)R_g(0)T_s/2$ and the average received symbol energy E_{av} is $A^2(1+\beta^2)R_g(0)T_s/4$. Since each symbol carries four bits of information, then the average received energy per bit E_b , is $E_{av}/4$.

The receiver block diagram for the k^{th} diversity branch is shown in Fig.3. To determine the amplitude encoded bit both $r_k(t)$ and a symbol delayed version, $r_k(t-T_s)$, are passed through an integrator and square-law detector. Diversity combining is performed by summing the outputs from all the diversity branches. The two decision variables $|U|^2$ and $|K|^2$ illustrated in Fig.3 are represented mathematically as follows:

$$|U|^2 = \sum_{k=1}^L |U_k|^2 = \sum_{k=1}^L \left| \int_0^{T_s} r_k(t) dt \right|^2 \quad (5)$$

$$|K|^2 = \sum_{k=1}^L |K_k|^2 = \sum_{k=1}^L \left| \int_0^{T_s} r_k(t-T_s) dt \right|^2 \quad (6)$$

A full description of the encoding and decoding process for 16-DAPSK can be found in references [2],[6]. For the amplitude encoded data bit the decision rule given below is used. If the following expression is satisfied then the data is received as a binary '1', otherwise a '0' is assumed.

$$\xi_L^2 < \frac{|U|^2}{|K|^2} < \xi_H^2 \quad (7)$$

In the above equation ξ_H and ξ_L represent the high and low decision thresholds, both of which are functions of the ring ratio β [2],[5]:

$$\xi_H = \frac{1+\beta}{2}, \quad \xi_L = \frac{2}{1+\beta} \quad (8)$$

For the phase encoded bits, conventional 8-DPSK employing postdetection maximum-ratio combining is assumed.

3 Error Probability Calculation

Since the receiver detects the amplitude and phase bits independently, the error probability for 16-DAPSK is determined by calculating the amplitude and phase error probabilities separately. These values are then combined to form the overall error probability for 16-DAPSK

Amplitude error probability: Assuming equally likely symbols, the high and low amplitudes will occur uniformly with a probability of 0.5. The amplitude error probability can be then written as follows:

$$P_a = \frac{1}{4} [P_a(HH) + P_a(LL) + P_a(HL) + P_a(LH)] \quad (9)$$

where $P_a(HL)$ represents the amplitude error probability for an amplitude sequence going from high to low.

$$\begin{aligned} P_a(HH) &= P[|U_H|/|K_H| > \xi_H] + P[|U_H|/|K_H| < \xi_L] \\ P_a(LL) &= P[|U_L|/|K_L| > \xi_H] + P[|U_L|/|K_L| < \xi_L] \\ P_a(HL) &= P[|U_L|/|K_H| > \xi_L] - P[|U_L|/|K_H| < \xi_H] \\ P_a(LH) &= P[|U_H|/|K_L| > \xi_L] - P[|U_H|/|K_L| < \xi_H] \end{aligned}$$

where $|U_H|$ and $|U_L|$ are used to denote the use of the outer and inner rings respectively. The terms $|U_i|/|K_j| > \xi_m$ and $|U_i|/|K_j| < \xi_m$ can be transformed into $|U_i|^2 - \xi_m^2|K_j|^2 > 0$ and $|K_j|^2 - \xi_m^2|U_i|^2 < \xi_m$ which are special cases of a general quadratic form in complex-valued Gaussian random variables. Using the

techniques proposed in [3], the error probability for each of the amplitude transients can be written as:

$$\begin{aligned} P_a(HH) &= B_1 + B_2 \\ P_a(LL) &= B_3 + B_4 \\ P_a(HL) &= B_5 - B_6 \\ P_a(LH) &= B_7 - B_8 \end{aligned} \quad (10)$$

where

$$B_i = \frac{1}{(1 + C_i)^{2L-1}} \sum_{k=0}^{L-1} \binom{2L-1}{k} C_i^k \quad (11)$$

with

$$\begin{aligned} C_1 &= [1 + L_a(\mathcal{A}\beta^2, \xi_H)] [1 - L_a(\mathcal{A}\beta^2, \xi_H)]^{-1} \\ C_2 &= [1 + L_b(\mathcal{A}\beta^2, \xi_L)] [1 - L_b(\mathcal{A}\beta^2, \xi_L)]^{-1} \\ C_3 &= [1 + L_a(\mathcal{A}, \xi_H)] [1 - L_a(\mathcal{A}, \xi_H)]^{-1} \\ C_4 &= [1 + L_b(\mathcal{A}, \xi_L)] [1 - L_b(\mathcal{A}, \xi_L)]^{-1} \\ C_5 &= [1 + L_c(\xi_L)] [1 - L_c(\xi_L)]^{-1} \\ C_6 &= [1 + L_c(\xi_H)] [1 - L_c(\xi_H)]^{-1} \\ C_7 &= [1 + L_d(\xi_L)] [1 - L_d(\xi_L)]^{-1} \\ C_8 &= [1 + L_d(\xi_H)] [1 - L_d(\xi_H)]^{-1} \\ \mathcal{A} &= 8\gamma_b(1 + \beta^2)^{-1} \end{aligned}$$

The equations for $L_i(\cdot)$ are defined in [7]. The amplitude error probability can now be determined by inserting the above expressions into (9).

Phase error probability: The symbol error probability (P_M) for M-ary DPSK in a slowly fading channel is given in [3] assuming L^{th} order diversity. The performance is entirely specified by the cross-correlation coefficient μ , between the delayed and present received signal. For 16-DAPSK, two amplitude levels are used (i.e. four different values of μ) with equal probability of occurrence. Hence, for L^{th} order diversity, the average phase error probability for 16-DAPSK is:

$$P_p = [P_8(\mu_{HH}) + P_8(\mu_{LL}) + P_8(\mu_{HL}) + P_8(\mu_{LH})] \quad (12)$$

where

$$\begin{aligned} \mu_{HH} &= \beta^2 \mathcal{A} \mu_g(T_s) (1 + \beta^2 \mathcal{A})^{-1} \\ \mu_{LL} &= \mathcal{A} \mu_g(T_s) (1 + \mathcal{A})^{-1} \\ \mu_{HL} &= \mu_{LH} \\ &= \beta \mathcal{A} \mu_g(T_s) (1 + \mathcal{A} + \beta^2 \mathcal{A} + \beta^2 \mathcal{A}^2)^{-1/2} \end{aligned}$$

Combined error probability: The symbol error probability P_s , can be calculated by inserting (9) and (12) into the following equation:

$$P_s = P_a + P_p - P_a P_p \quad (13)$$

Assuming Gray coding to be used, the average bit error probability P_b , can be written as below:

$$P_b \approx P_s/4 \quad (14)$$

From (9) and (12), the error probability for 16-DAPSK can be seen to be a function of the average received E_b/N_o per channel γ_b , the ring ratio of the modulation scheme β , the two decision thresholds ξ_H , ξ_L , and the order of diversity L .

4 Results

For 16-DAPSK the importance of the ring ratio was determined with both 1st and 2nd order diversity combining. For both cases an ideal slow fading Rayleigh channel was assumed (i.e. $f_d T_s = 0$). Fig.4 and 5 show that a ring ratio of around 2 is optimal for both cases and that this value is not sensitive to the order of diversity.

Assuming this optimum ring ratio, Fig.6 shows the average bit error probability for 1st and 2nd order diversity. Results for both $f_d T_s = 0$ and $f_d T_s = 0.01$ have been included together with those obtained through computer simulation. This graph clearly shows the limiting effect of random FM and the gains achieved through 2nd order diversity.

To further investigate the impact of random FM the resulting irreducible error probability was determined for a range of $f_d T_s$. These results are shown in Fig.7 together with those obtained for QDPSK. For both systems the addition of diversity results in a significant improvement allowing far higher mobile speeds to be tolerated.

5 Conclusions

Although QDPSK can tolerate higher values of random FM, it is important to remember that 16-DAPSK achieves twice the spectral efficiency. For systems where the channel varies slowly relative to the data rate, 16-DAPSK would clearly be preferred due to its increased spectral efficiency. The results also indicate that for channels limited by random FM, the application of diversity to 16-DAPSK results in a performance that is superior to that achieved by single order QDPSK. This implies that for channels suffering random FM, the addition of diversity is more important than the particular choice of modulation scheme.

16-DAPSK has been shown to be a robust, spectrally efficient technique for operating in mobile fading chan-

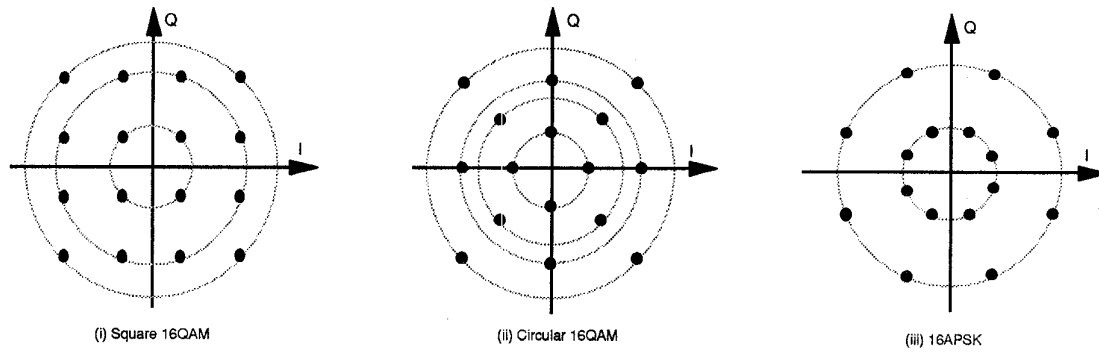


Fig.1: Signal constellations for 16-level modulation

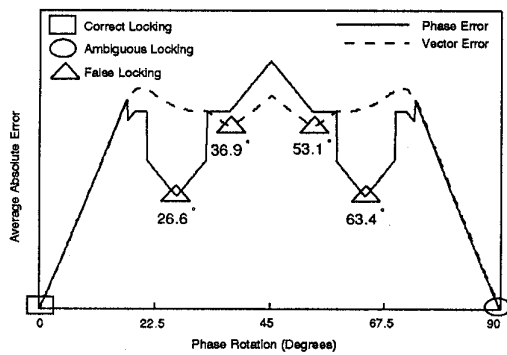


Fig.2: False Locking for Square 16QAM

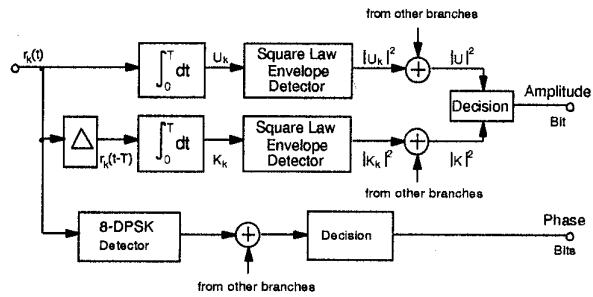


Fig.3: Receiver block diagram at K^{th} diversity branch

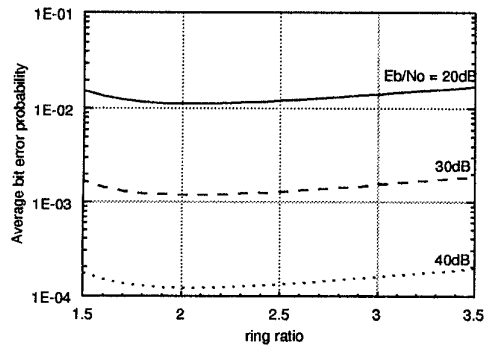


Fig.4: Error probability vs. ring ratio ($L=1$, $f_0T_s=0$)

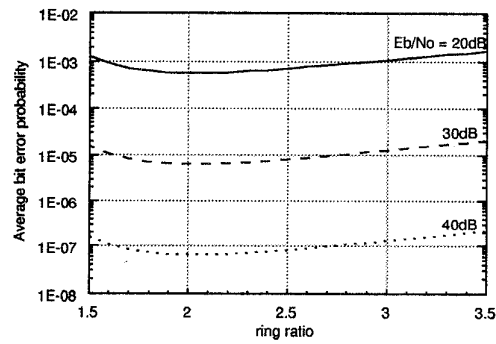


Fig.5: Error probability vs. ring ratio ($L=2$, $f_0T_s=0$)

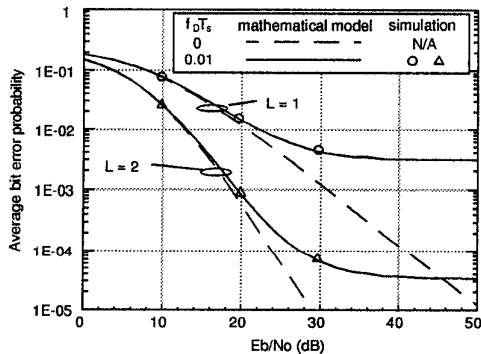


Fig.6: Performance of 16DPSK signal with diversity

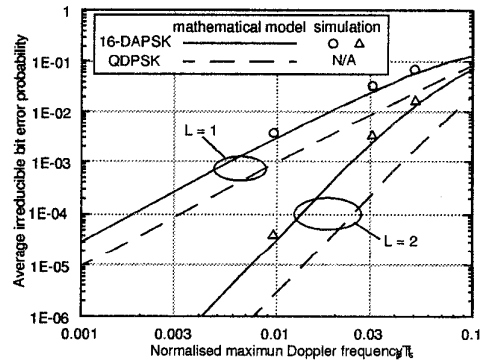


Fig.7: Average irreducible bit error probability vs. f_0T_s

nels. The use of differential detection not only allows simple reception but also avoids the complexities of carrier recovery.

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